Effects of Transpiration on Flow Past Impulsively Started Plate

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INTRODUCTION

In many industrial applications, the flow past an infinite vertical plate, started impulsively from rest, plays an important role, e.g., flow past a radiator. Stokes (1851) studied the flow of a viscous fluid past an infinite horizontal plate started impulsively in its own plane. Such a flow past an infinite isothermal vertical plate, whose temperature differs from that of the ambient fluid, was studied first by Soundalgekar (1977), taking into account the presence of free convection current. The corresponding flow problem for permeable plate has been studied by Soundalgekar et al. (1981), where the effects of suction or injection and free convection currents on velocity fields were presented. In many practical situations the plate temperature is not isothermal and generally the plate is cooled or heated with constant heat flux. Recently, Revankar (1982) has studied the effects of free convection currents on the velocity field near the boundary layer of an impulsively started permeable vertical plate with constant heat flux. This note will discuss the effects of transpiration on the flow past an impulsively started vertical plate cooled or heated with constant heat flux. It is further assumed that the plate is uniformly permeable and the same fluid transfers at a uniform and constant rate through the wall. Such a physical situation has not been studied in the literature, hence it is presented here.

ANALYSIS

The governing equations for the situation considered are (Soundalgekar et al., 1981)

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$P\frac{\partial \theta}{\partial t} + vP\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \tag{2}$$

The initial and boundary conditions for the present problem are

$$u = 0, \quad \theta = 0 \quad \text{at } y = 0\ (t \leqslant 0)$$

$$\left.\begin{array}{lll} u = 1, & \dfrac{\partial\theta}{\partial y} = -1 & \text{at } y = 0 \\ u = 0, & \theta = 0 & \text{as } y \to \infty \end{array}\right\}(t > 0) \tag{3}$$

For small suction or injection, $v \ll 1$, we expand the velocity and temperature up to first order as

$$\begin{aligned} u &= u_0(y,t) + vu_1(y,t) \\ \theta &= \theta_0(y,t) + v\theta_1(y,t) \end{aligned} \tag{4}$$

Hence the solutions obtained by using Laplace-transform technique are as follows:

$$\theta = 2(1 + vP\eta\sqrt{t})\sqrt{t}\left[\frac{\bar{e}^{\eta^2 P}}{\sqrt{\pi P}} - \eta \operatorname{erfc}(\eta\sqrt{P})\right] + \frac{vt}{2}\left[\operatorname{erfc}(\eta\sqrt{P})(1 + 2\eta^2 P) - \frac{2\eta\sqrt{P}}{\sqrt{\pi}}\bar{e}^{\eta^2 P}\right] \tag{5}$$

$$\begin{aligned} u = {} & (1 + v\eta\sqrt{t})\operatorname{erfc}(\eta) + (4t)^{3/2}\frac{G}{\sqrt{P}(P-1)}(1 + v\eta\sqrt{t})\left[\frac{\bar{e}^{\eta^2}}{6\sqrt{\pi}}(1 + \eta^2) \right. \\ & \left. - \eta\operatorname{erfc}(\eta)\left(\frac{1}{4} + \frac{\eta^2}{6}\right)\right] - (4t)^{3/2}\frac{G}{\sqrt{P}(P-1)}\left(1 + \frac{vP}{2}\right)\left[\frac{\bar{e}^{\eta^2 P}}{6\sqrt{\pi}}(1 + \eta^2 P) \right. \\ & \left. - \eta\sqrt{P}\operatorname{erfc}(\eta\sqrt{P})\left(\frac{1}{4} + \frac{\eta^2 P}{6}\right)\right] + \frac{vGt^2}{2(P-1)}\left[\left(\frac{2}{3}\eta^4 + 2\eta^2 + \frac{1}{2}\right)\operatorname{erfc}(\eta) \right. \\ & \left. - \frac{\eta\bar{e}^{\eta^2}}{3\sqrt{\pi}}(5 - 2\eta^2) - \left(\frac{2}{3}\eta^4 P^2 + 2\eta^2 P + \frac{1}{2}\right)\operatorname{erfc}(\eta\sqrt{P}) + \frac{\eta\sqrt{P}\bar{e}^{\eta^2 P}}{3\sqrt{\pi}}(5 - 2\eta^2 P)\right] \end{aligned} \tag{6}$$

for $P \neq 1$, while for $P = 1$

$$\begin{aligned} u = {} & (1 + v\eta\sqrt{t})\operatorname{erfc}(\eta) \\ & -(4t)^{3/2}\frac{G\eta}{2}\left\{\frac{\bar{e}^{\eta^2}}{3\sqrt{\pi}}\left[(\eta^3 + \eta^2 - 3\eta + 1)\frac{v}{2} + \eta^3 - 3\eta\right] \right. \\ & \left. + \frac{\operatorname{erfc}(\eta)}{4}\left(1 - \frac{v}{4} + \frac{\eta^3 v}{3} - 2\eta^3\right)\right\} - \frac{vGt^2}{2}\left[\operatorname{erfc}(\eta)\left(\frac{4}{3}\eta^4 + 2\eta^2\right) \right. \\ & \left. - \frac{\bar{e}^{\eta^2}}{\sqrt{\pi}}\left(\frac{2}{3}\eta^5 - \frac{2}{3}\eta^4 - \frac{8}{3}\eta^3 - 2\eta^2 + \frac{5}{6}\eta - \frac{1}{2}\right)\right] \end{aligned} \tag{7}$$

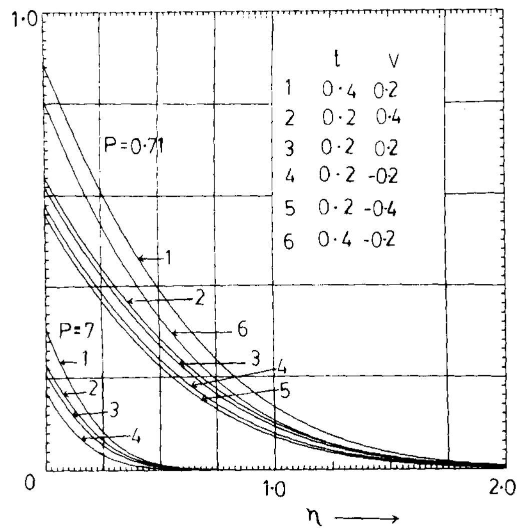

Figure 1. Temperature profile for air and water.

Here $\eta = y/2\sqrt{t}$. To get physical insight into the problem, we have computed the numerical values of u and θ for different values of t, G, and v. Now $v < 0$ represents suction and $v > 0$ represents injection. The physical meaning of G depends on the value of q. If $q > 0$, then $G > 0$, and hence the free convection currents travel from the plate to the fluid, which is physically equivalent to cooling of the plate. The case $G < 0$ corresponds to heating of the plate by free convection currents. In Figure 1 the velocity profiles in the boundary layer are presented for air ($P = 0.71$) and water ($P = 7.0$) for different values of suction and injection at the plate. The temperature of the fluid decreases with increasing Prandtl number. With increase in suction at the plate, the fluid temperature decreases, whereas in the presence of injection ($v > 0$), the temperature increases with increase in injection velocity.

In Figure 2 the velocity profiles are presented for air for the case of cooling ($G > 0$) and heating ($G < 0$) of the plate. Higher cooling of the plate or an increase in injection at the plate causes an increase in fluid velocity. An increase in the suction velocity decreases the fluid velocity in the case of the plate being cooled by free convection currents. In the case of heating of the plate, increase in injection velocity enhances the velocity of the fluid. Thus the effects of suction or injection on the velocity field are of opposite nature for both cooling and heating modes.

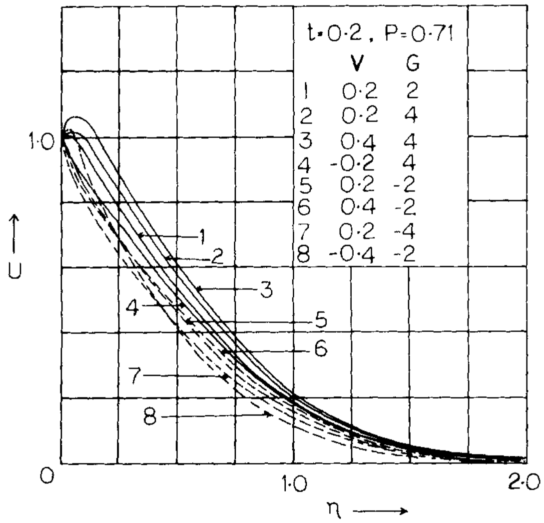

Figure 2. Velocity profile for air.

TABLE 1. VALUES OF SKIN FRICTION

t	v	G/P	0.71	7.0
0.2	0.2	2	1.2151	1.2348
0.2	0.4	2	1.4263	1.5121
0.4	0.2	2	0.6127	0.7452
0.2	−0.2	2	0.7927	1.2053
0.2	−0.4	2	0.5815	1.2836
0.2	0.2	−2	1.5079	1.4883
0.2	0.4	−2	1.4967	1.3635
0.4	0.2	−2	1.3713	1.2389
0.2	−0.2	−2	1.5303	1.1177
0.2	−0.4	−2	1.5415	1.1263

After knowing the velocity field it is interesting to study the skin friction at the plate, which is given in nondimensional form as

$$\tau = -\left.\frac{\partial u}{\partial y}\right|_{y=0} = -\left.\frac{1}{2\sqrt{t}}\frac{\partial u}{\partial \eta}\right|_{\eta=0} \tag{8}$$

Hence from Eqs. 6 and 8, we have the skin friction given by

$$\tau = \frac{1}{\sqrt{\pi t}} - Gt\left[\frac{1}{\sqrt{P}(\sqrt{P}+1)} + \frac{vP}{(P-1)}\right] + \frac{v}{2} + \frac{2vGt^{3/2}}{3}\left[\frac{1}{(\sqrt{P}+1)} + \frac{1}{\sqrt{P}(P-1)}\right] \tag{9}$$

The numerical values of τ are entered in Table 1. We observe that for $G > 0$, with increase in injection velocity at the plate, the skin friction increases, whereas for $G < 0$, the skin friction decreases for both air and water. But in case of suction at the plate, exactly opposite behaviors are observed. For the heating mode, the values of skin friction are higher in case of air compared to water. With increase in time, the skin friction decreases in all cases.

CONCLUSION

An analysis is carried out to study the effect of transpiration on the flow past an impulsively started vertical plate, cooled, or heated with constant heat flux. The effect of injection at the plate is found to increase temperature and velocity field in the boundary layer for both fluids studied (air and water). The solution presented for the problem is exact and hence validates its use for most engineering applications.

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NOTATION

C_p = specific heat at constant temperature
erfc = complementary error function
g = acceleration due to gravity
G = Grashof number, $\nu^2 g\beta q/kU_0^4$
k = thermal conductivity
P = Prandtl number, $\nu\rho C_p/k$

q = heat flux
T' = temperature of the fluid near the plate
T'_∞ = temperature of the fluid at infinity
t = dimensionless time, $t'U_0^2/\nu$
t' = time
u' = velocity of the fluid in the x direction
u = dimensionless velocity of the fluid in the x direction, u'/U_0
U_0 = velocity of plate
v' = velocity of the fluid in y direction
v = dimensionless velocity of the fluid in y direction, v'/U_0
x',y' = coordinates along and normal to the plate
x,y = dimensionless coordinates, $y = y'U_0/\nu$

Greek Letters

ν = kinematic viscosity of the fluid
β = coefficient of volume expansion
ρ = density
θ = nondimensional temperature, $(T' - T'_\infty)/(q\nu/kU_0)$
τ' = skin friction
τ = dimensionless skin friction, $\tau'/\rho U_0^2$

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